

Exercise 8.2:

A viewer is in the point $(2, 1, 4)$ and looks at the (y, z) -plane which is illuminated by a directional light source. The light source lies in infinite distance in the direction of the vector $(1, 2, 3)$. At which point on the plane can the viewer see specular reflection?

Solution (sketch):

The point of specular reflection must be of the form $(0, y, z)$. The coordinates y and z are to be determined.

Normal vector to the (y, z) -plane: $(1, 0, 0)^\top$.

The vector connecting $(0, y, z)$ and the viewer: $\mathbf{v} = (2, 1 - y, 4 - z)^\top$.

”Reflection” of this connecting vector at the normal vector results in the direction of light that will cause specular reflection.

Difference vector between the connecting vector and the normal vector scaled to the same height ($x = 2$) as the connecting vector:

$$\mathbf{s} = \begin{pmatrix} 0 \\ y - 1 \\ z - 4 \end{pmatrix}.$$

Reflection of the connecting vector:

$$\tilde{\mathbf{v}} = \mathbf{v} + 2\mathbf{s} = \begin{pmatrix} 2 \\ y - 1 \\ z - 4 \end{pmatrix}.$$

This vector must point into the direction of the light source, i.e. there must be a constant $\lambda \in \mathbb{R}$ satisfying

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \lambda \tilde{\mathbf{v}} = \begin{pmatrix} 2\lambda \\ (y - 1)\lambda \\ (z - 4)\lambda \end{pmatrix}.$$

From this equation, we derive $\lambda = \frac{1}{2}$ and therefore $y = 5, z = 10$.

The viewer can see (ideal) specular reflection in the point $(0, 5, 10)$.