**Exercise 3.1:**

Derive the midpoint algorithm for drawing lines with a slope between $-1$ and $0$.

**Solution (sketch):**

**Case 1:** $E$, i.e. $(x_{p+1}, y_{p+1}) = (x_p + 1, y_p)$ was the pixel drawn after $(x_p, y_p)$.

$$d_{\text{new}} = F\left(x_p + 2, y_p - \frac{1}{2}\right) = dy \cdot (x_p + 2) - dx \cdot \left(y_p - \frac{1}{2}\right) + C$$

$$d_{\text{old}} = F\left(x_p + 1, y_p - \frac{1}{2}\right) = dy \cdot (x_p + 1) - dx \cdot \left(y_p - \frac{1}{2}\right) + C$$

$$\Delta_E = d_{\text{new}} - d_{\text{old}} = dy.$$

**Case 2:** $SE$, i.e. $(x_{p+1}, y_{p+1}) = (x_p + 1, y_p - 1)$ was the pixel drawn after $(x_p, y_p)$.

$$d_{\text{new}} = F\left(x_p + 2, y_p - \frac{3}{2}\right) = dy \cdot (x_p + 2) - dx \cdot \left(y_p - \frac{3}{2}\right) + C$$

$$\Delta_{SE} = d_{\text{new}} - d_{\text{old}} = dy + dx$$

$$\begin{aligned}
d_{\text{init}} &= F\left(x_0 + 1, y_0 - \frac{1}{2}\right) \\
&= dy \cdot (x_0 + 1) - dx \cdot \left(y_0 - \frac{1}{2}\right) + C \\
&= dy \cdot x_0 - dx \cdot y_0 + C + dy + \frac{dx}{2} \\
&= F(x_0, y_0) + dy + \frac{dx}{2} \\
&= dy + \frac{dx}{2}
\end{aligned}$$

Consider $D = 2 \cdot d$ for integer arithmetics.

$$\begin{aligned}
D_{\text{init}} &= 2 \cdot dy + dx \\
D_{\text{new}} &= D_{\text{old}} + \Delta \quad \text{where} \\
\Delta &= \begin{cases} 2 \cdot dy & \text{if } D_{\text{old}} > 0, \\ 2 \cdot (dy + dx) & \text{if } D_{\text{old}} < 0. \end{cases}
\end{aligned}$$